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Anelastic and elastic properties of a synthetic monocrystal of bismuth germanate $Bi_4Ge_3O_{12}$ at low temperatures

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Abstract

In view of the possible use of bismuth germanate (BGO, $Bi_4Ge_3O_{12}$) as a multicrystal detector of sonic pulses due to gravitational waves, the viscoelastic properties of a synthetic sample of BGO were studied. The Young modulus and internal friction of a synthetic monocrystal of BGO, were measured in the temperature range 10–300 K. On the basis of these results, the prospective use of a BGO detector is discussed.

1. Introduction

There is a growing interest in the use of synthetic bismuth germanate ($Bi_4Ge_3O_{12}$, BGO) in particle physics, in medical tomography and in astrophysics. At present the main applications are related to its scintillation properties, which have been studied in depth. Cubic type crystals, such as BGO, are generally the easiest crystals to grow and lend themselves to various established and well described techniques. Moreover, applications in particle physics have required the use of large crystals in quantities up to 10^4 pieces with a total mass of the order of several tons, as for the electromagnetic calorimeter of the L3 experiment at LEP [1]. For these reasons new improvements have been made in the purification and growth techniques.

Large masses are also used in the attempt to detect gravitational waves in the region of acoustic frequencies [2]. Most of these detectors consists of a massive solid cylinder made of aluminium, with a fundamental longitudinal frequency in the 1 kHz region and, at present, some of them operate in coincidences over a long time period. The principle of operation of these antennas is based on the detection of a sudden change in the vibration state of the fundamental vibration mode. The integrated cross-section Σ for the absorption of gravitational wave energy is a linear function of the Young modulus of the antenna material Y:

$$\Sigma = \frac{32GV}{15\pi}\frac{Y}{c^3} \qquad (1)$$

where V is the antenna volume, G is the gravitational constant and c the velocity of light. This formula holds if the average over the possible directions and polarization status of the gravitational wave is considered. The expected signal is very weak and it must be detected against the thermal brownian noise of the detector and the electronic noise associated with the readout circuit. The energy absorbed from the incoming gravitational wave by a resonant mode of oscillation of a solid body must be larger than the energy fluctuation ΔE in the observational interval of time Δt:

$$\Delta E = 4k_B T_n\left(\frac{1}{\lambda} + \frac{T}{T_n}\frac{1}{\beta_Q}\right)^{1/2} \qquad (2)$$

where Q is the overall quality merit factor of the first longitudinal vibration, T_n the noise temperature of the electronic amplifier, λ is the ratio of the transducer impedance and the noise impedance of the amplifier, and the factor β takes into the account the conversion efficiency of the vibration energy of the antenna in the readout system.

ΔE can be decreased by lowering the temperature of the antenna and using materials with low intrinsic dissipation (high Q values). Thus, the low dissipation of the material is a condition for reducing detector noise, while a high value of Young's modulus increases the signal in the antenna.

In view of possible use of BGO crystals as gravitational wave detectors, we studied their elastic properties in the temperature range 10–300 K by measuring the quality factors and the resonance frequencies of the odd longitudinal vibration modes of a cylindrical sample. On the basis of the results the feasibility of a BGO gravitational wave detector is discussed.

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2. Experimental method

The mechanical properties of the scintillating crystals have not been investigated in depth. These crystals are in general brittle and the classical tension–compression measurements that are standard for metals, cannot be obtained and, in this case, the elastic values are deduced from results of the modulus of rupture [3]. This destructive method has a limited accuracy and the implementation of this method at low temperature is far from easy. We apply the resonance method, and we derive the elastic properties of the BGO by measuring the characteristics of the longitudinal vibration modes of a sample.

Our sample of synthetic BGO is a cylinder of length 0.201 m, of diameter 0.02 m and mass 0.4 kg. It was suspended on a small aluminium 5056 fork and the zone of contact between the suspension and the sample lies on the lateral surface of the crystal at its mass centre section. We excited the vibrations of the small BGO bar using a small piezoelectric ceramic glued on the base of the aluminium suspension. The vibration of the BGO bar was detected by a second piezoelectric ceramic glued on the centre of the crystal. The frequencies were measured with a resolution of 0.01 Hz by driving the ceramic glued on the suspension with a voltage having a white noise power spectrum in the frequency range of interest and by analysing the signal output of the piezoelectric transducer with an Hp-35660A dynamic signal analyser. The quality factor was derived by measuring the free decay time of the resonance mode after having switched off the excitation. By means of a calibrated silicon diode in thermal contact with the suspension, the temperature of the sample was monitored with an accuracy of 0.5 K. The crystal was located inside a vacuum chamber made of stainless steel tightened with an indium gasket. During measurements the residual pressure inside the chamber was in the region of 10^{-4} Pa. This chamber was located inside a vertical liquid helium cryostat having an inner diameter of 0.35 m.

Let us discuss the limits of this method by evaluating the influence of the transducer glued on the BGO sample on the measured frequencies and quality factors. Assuming good mechanical contact between the transducer and the vibrating body, it can be shown [4] that the overall resonance frequency of the system ν is

$$\nu^2=\nu_s^2\left(1+2\,\frac{V_{pz}Y_{pz}}{V_sY_s}\right)\left(\frac{1+V_{pz}Y_{pz}C_{pz}}{1+m_{pz}/m_s}\right) \tag{3}$$

where m_{pz}, m_s, V_{pz}, V_s, Y_{pz} and Y_s are the masses, the volumes and Young's moduli of the transducer and of the sample of BGO, C_{pz} is the electric capacitance of the piezoelectric ceramic and ν_s is the unperturbed resonance frequency of the sample. In this experimental configuration the perturbation effect estimated on the basis of this formula is of the order of $1/10^6$ because we have $2V_{pz}/V_s\approx10^{-5}$, $m_{pz}/m_s\approx10^{-3}$ and $C_{pz}\approx100$ pF at low temperatures.

A similar relation holds for the measured quantity Q

$$Q^{-1}=Q_s^{-1}+Q_{pz}^{-1}\,2\,\frac{V_{pz}}{V_s}+\beta Q_{el}^{-1} \tag{4}$$

where Q_s, Q_{pz} are the quality factors of the sample and of the output transducer, β is the ratio of the electrical energy in the piezoelectric ceramic to the total mechanical energy in the system and Q_{el} is its electrical quality factor. In the present experimental set-up βQ_{el}^{-1} is negligible because β is of the order of 10^{-7} and at low temperatures we measured $Q_{el}\approx10^4$. The contribution of the mechanical losses in the transducer to the overall Q of the system is given by the term $Q_{pz}^{-1}2V_{pz}/V_s$. In this experimental configuration Q_{pz} is of the order of 100. As a consequence the effect of transducer perturbation on the Q measurements is relevant for values up to 10^7.

3. Experimental results

We identified the resonance frequencies of the first, third and fifth longitudinal vibration modes of the antenna at $T=295.5$ K, by comparing the sequence of measured values

$\nu_1=9612.51$ Hz $\quad \nu_3=28806.51$ Hz $\quad \nu_3=47839.14$ Hz

with the sequence expected on the basis of the zero-order frequency equation of the longitudinal vibration of a cylinder

$$\nu_n=n\,\frac{1}{2L}\left(\frac{Y_s}{\rho_s}\right)^{1/2}$$

where L is the length of the sample and $\rho_s=7130$ kg m^{-3} its density, which we measured at room temperature.

In Fig. 1 we report the measurements of the frequencies of the three modes *vs.* temperature. The frequency change is directly related to the variations in the Young modulus with the temperature. In fact, the change in the geometrical dimensions of the sample with temperature is not significant because the coefficient of thermal expansion decreases with the temperature and for BGO at room temperature [5] it is of the order of 5×10^{-6} K^{-1}.

The Poisson ratio is derived using the frequency equation of the longitudinal vibration of a rod of radius R and length L, obtained by Pochammer [6] in a second-

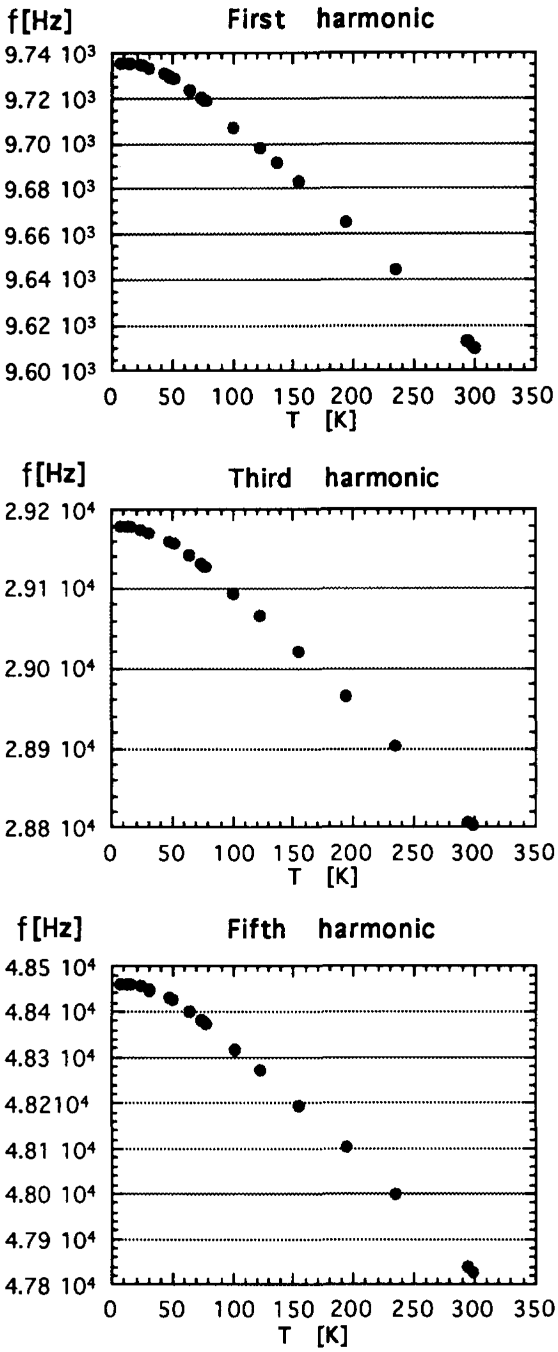

Fig. 1. The frequencies of the three longitudinal vibration modes of the BGO cylinder *vs.* the temperature.

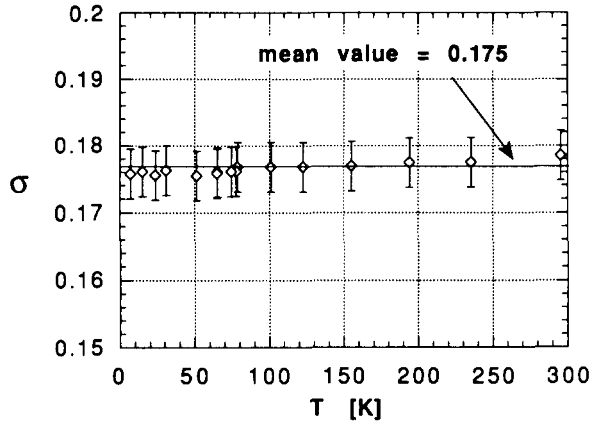

Fig. 2. Experimental data of the BGO Poisson ratio *vs.* the temperature.

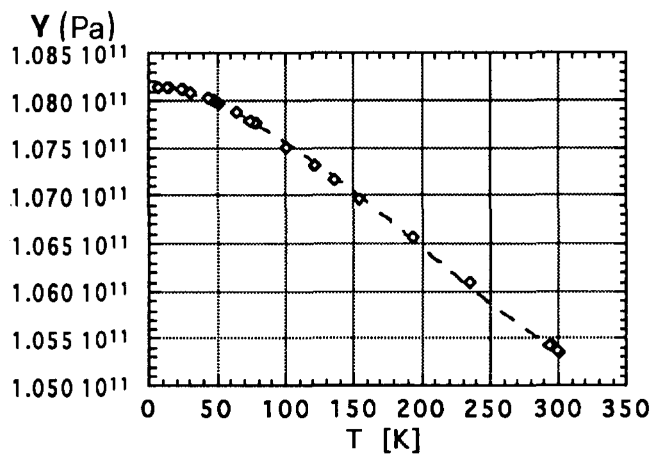

Fig. 3. The Young modulus *vs.* the temperature of the BGO. The line is obtained with a polynomial regression of second order (see text).

order approximation:

$$\nu_n = n\frac{1}{2L}\left\{\frac{Y}{\rho_s}\left[1-\sigma^2\left(\frac{n\pi R}{L}\right)^2\right]\right\}^{1/2} \qquad (5)$$

Then, from the frequency data of the first and the fifth harmonics as functions of the temperature, we obtain the temperature dependence of the Poisson ratio, shown in Fig. 2.

Then we derive the dependence of the Young modulus on temperature, shown in Fig. 3, by normalizing the frequency data of the harmonics of different order. In the same figure, we show the result of the data fit performed with a second-order regression line of equation

$$Y=1.082\times10^{11}-2.380\times10^{6}\ T-4.647\times10^{4}\ T^2+76.8T^3 \qquad (6)$$

where the Young modulus Y and the temperature T are expressed in Pascals and in degrees Kelvin respectively. Then, we find that the Young modulus of synthetic BGO crystal attains a minimum value of $Y=1.082\pm0.006$ Pa at low temperatures.

We also measured the Q factor of the small resonator at the first longitudinal mode. It is well known that relaxation studies in the kilohertz range of frequencies give information on the atomic migration and/or reorientation processes, while at higher frequencies the investigations are related to electron and phonon relaxation processes. As in the case of some crystals and metals, the relaxation processes in our sample are "frozen" on lowering the temperature. In fact, we

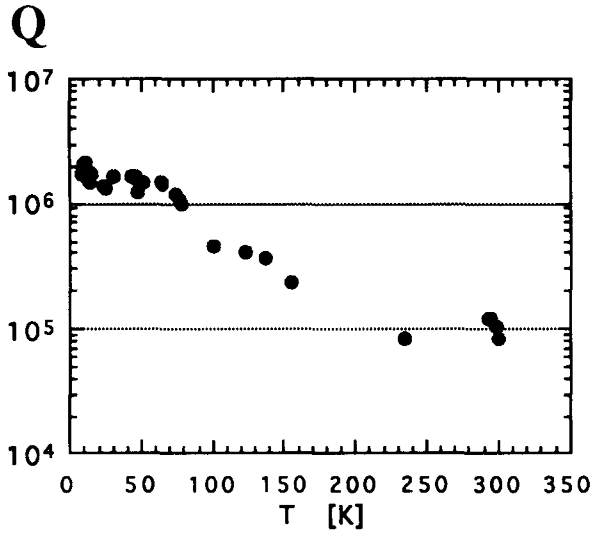

Fig. 4. The quality merit factor of the first longitudinal vibration mode *vs.* the temperature.

observe that the quality factor of the BGO increases as the temperature decreased, as is shown in Fig. 4, attaining the maximum value of $Q \approx 2.13 \times 10^6$ at 10 K. We checked that this value is not limited by the suspension losses by measuring the dissipation factor of the nearest vibration mode of the suspension coupled to the longitudinal mode of the sample. Taking into account the frequency distance of the two modes (9 kHz) and the quality factor of the suspension mode ($Q_s = 1000$) we conclude that the influence of the suspension on the measurement of the quality factor of the sample is negligible [7].

4. Final remarks

As shown in eqn. (1), the cross-section depends on the total volume of the antenna, and in principle the detector mass can be distributed in several smaller pieces, cooled separately and distributed over a wide area. This modular antenna has some obvious advantages.

The reliability of the apparatus increases by a factor equal roughly to the number of the elements, because the motion of each mass is detected separately and the cooling time is not set by the total mass, but by the mass of each element.

Moreover, the background signal due to the effect of cosmic rays interacting with the detector is easily removed with normal anticoincidence techniques between the various elements, while the statistical confidence level for the weak gravitational signals is increased with coincidence techniques between the elements. Finally, we can easily obtain a large detector bandwidth with a suitable spread of fundamental frequency values of the different pieces.

The good anelastic behaviour and relatively high density (with respect to aluminium) of BGO monocrystals make them possible candidates as elements of such a modular antenna.

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